

In a nutshell: Approximating values using interpolating polynomials

In this topic, we assume that the x values are equally spaced. We will assume that equal spacing equals h .

Two points centred

Given the points (x_{k-1}, y_{k-1}) , (x_k, y_k) , approximate the value at $x = \frac{x_{k-1} + x_k}{2} + \delta h$ where $-0.5 < \delta < 0.5$ using

$$y_{k-1} + (\delta + 0.5)(y_k - y_{k-1}).$$

Three points centred

Given the points (x_{k-1}, y_{k-1}) , (x_k, y_k) , (x_{k+1}, y_{k+1}) approximate the point $x = x_k + \delta h$ where $-0.5 < \delta < 0.5$ using

$$\left((y_{k+1} - 2y_k + y_{k-1})\delta + (y_{k+1} - y_{k-1}) \right) \frac{\delta}{2} + y_k.$$

Four points centred

Given the points (x_{k-2}, y_{k-2}) , (x_{k-1}, y_{k-1}) , (x_k, y_k) , (x_{k+1}, y_{k+1}) , approximate the value at $x = \frac{x_{k-1} + x_k}{2} + \delta h$ where $-0.5 < \delta < 0.5$ using

$$\left(\left(\frac{y_{k+1} - 3y_k + 3y_{k-1} - y_{k-2}}{6} \delta + \frac{y_{k+1} - y_k - y_{k-1} + y_{k-2}}{4} \right) \delta + \frac{-y_{k+1} + 27y_k - 27y_{k-1} + y_{k-2}}{24} \right) \delta + \frac{-y_{k+1} + 9y_k + 9y_{k-1} - y_{k-2}}{16}.$$

Two points backward (same as two points centred)

Given the points (x_{k-1}, y_{k-1}) , (x_k, y_k) , approximate the value at $x = \frac{x_{k-1} + x_k}{2} + \delta h$ where $-0.5 < \delta < 0.5$ using

$$y_{k-1} + (\delta + 0.5)(y_k - y_{k-1}).$$

Three points backward

Given the points (x_{k-2}, y_{k-2}) , (x_{k-1}, y_{k-1}) , (x_k, y_k) approximate the point $x = \frac{x_{k-1} + x_k}{2} + \delta h$ where $-0.5 < \delta < 0.5$ using

$$\left(\frac{y_k - 2y_{k-1} + y_{k-2}}{2} \delta + (y_k - y_{k-1}) \right) \delta + \frac{3y_k + 6y_{k-1} - y_{k-2}}{8}.$$

Four points backward

Given the points (x_{k-3}, y_{k-3}) , (x_{k-2}, y_{k-2}) , (x_{k-1}, y_{k-1}) , (x_k, y_k) approximate the point $x = \frac{x_{k-1} + x_k}{2} + \delta h$ where $-0.5 < \delta < 0.5$ using

$$\left(\left(\frac{y_k - 3y_{k-1} + 3y_{k-2} - y_{k-3}}{6} \delta + \frac{3y_k - 7y_{k-1} + 5y_{k-2} - y_{k-3}}{4} \right) \delta + \frac{23y_k - 21y_{k-1} - 3y_{k-2} + y_{k-3}}{24} \right) \delta + \frac{5y_k + 15y_{k-1} - 5y_{k-2} + y_{k-3}}{16}$$